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Measurements of a laminar wake in a confined stratified fluid

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Two-dimensional upstream and downstream wakes are produced by extremely slowly moving cylinders in a tank containing a linearly stratified salt solution. The upstream and downstream wakes appear to be of comparable strength, consistent with the symmetry predicted by theories that take account of density diffusion as well as viscosity. This contrasts with the quasi-non-diffusive régime recently studied experimentally by Browand & Winant (1972). The present velocity profiles are essentially similar to those predicted by diffusion theories, although the magnitudes of the velocities appear to be greatly attenuated by the proximity of the upstream and downstream walls.

Preliminary experiments on the flow produced by a cylinder moving horizontally normal to its horizontal longitudinal axis in a density stratified fluid are described. The Reynolds numbers based on the body velocity U and the body cross-sectional height $2b$ are approximately 1 and the Richardson numbers or internal Froude numbers b^2g/U^2h are about 10^4, where h is the stratification scale $\rho_\infty(\partial\rho_\infty/\partial z)^{-1}$. The Schmidt number ν/D is approximately 10^3, where ν is the viscosity and D is the salt diffusivity.

A glass-sided tank 1·8 m long, 0·9 m high and 0·55 m wide is filled with sodium chloride solution having a constant density gradient. A cylinder which completely spans the tank is supported by a vertical strut attached to a hydraulic drive on the base of the tank. Dye particles are dropped through the brine solution producing very stable dye lines which remain stationary relative to the surrounding fluid and are visible for up to 25 min. A beam of light parallel to the axis of the cylinder passes through the working section and throws shadows of the dye lines on to a screen where they are recorded. Dye line profiles at successive times are shown in figures 1 and 2, and examples of the variation in velocity at the centre-line of the upstream and downstream wakes are shown in figure 3. The kinematic viscosity is $1{\cdot}15\,\mathrm{mm^2\,s^{-1}}$ and the salt diffusivity is $1{\cdot}1\times10^{-3}\,\mathrm{mm^2\,s^{-1}}$. The asymmetry of the profiles is due to a slight movement in the ambient fluid. Experiment (*b*) in figure 3 corresponds to one set of results (indicated by the crosses) in which the centre-line velocity ahead of the model falls rapidly at first, and it was thought that these results were not being affected too much by the upstream wall.

At the highest cylinder velocity which is $0{\cdot}07\,\mathrm{mm\,s^{-1}}$ (not shown) or with the largest cylinder of 25·4 mm diameter, or when the cylinder is close to an end wall,

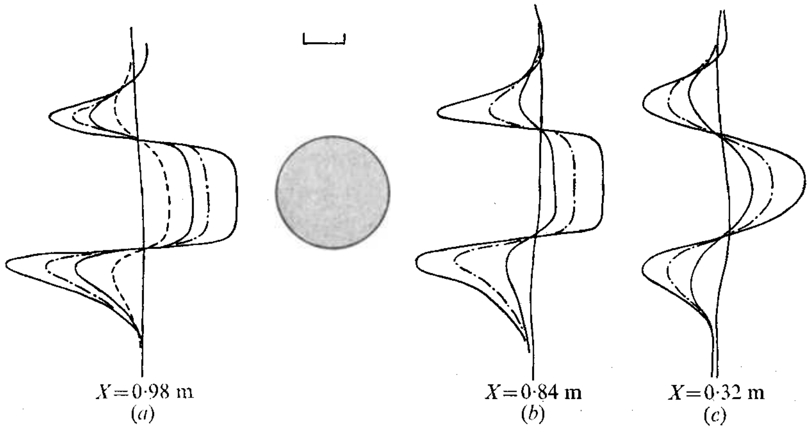

FIGURE 1. Dye-line drift profiles when a circular cylinder of diameter 25·4 mm starts to move at time $t = 0$ with a velocity of 0·032 mm s^{-1}. The cylinder is initially at the position $X = 0{\cdot}88$ m. X is measured from the upstream wall. The scale mark is of length 10 mm. (*a*) Drift lines are at times $t = 0$, 240, 415, 550 and 710 s. (*b*) $t = 0$, 180, 350 and 600 s. (*c*) $t = 0$, 500, 825 and 1110 s. The stratification scale height h is 5·5 m.

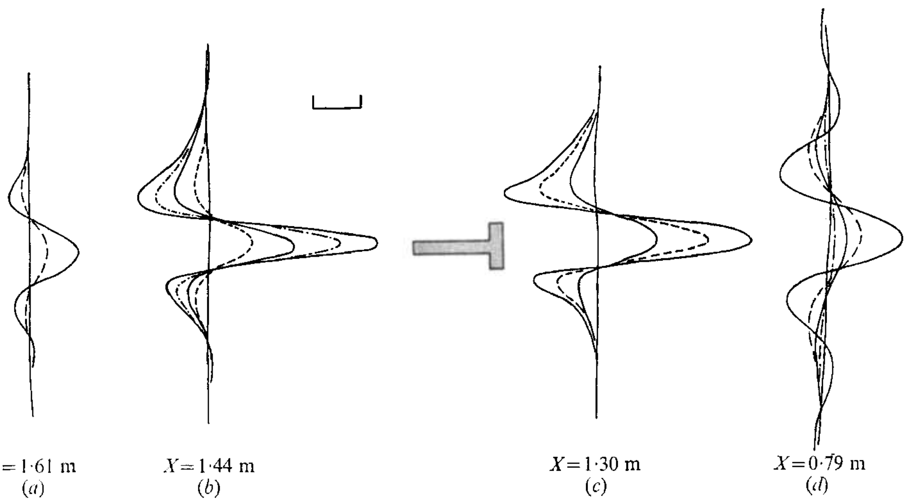

FIGURE 2. Dye-line drift profiles when a two-dimensional tee-shaped body of height 10 mm starts to move at time $t = 0$ with a velocity of 0·060 mm s^{-1}. The body is initially at the position $X = 1{\cdot}37$ m. The scale mark is length 10 mm. (*a*) $t = 0$, 280 and 620 s. (*b*) $t = 0$, 230, 390, 560 and 700 s. (*c*) $t = 0$, 330, 510 and 700 s. (*d*) $t = 0$, 180, 420, 600 and 1350 s. The stratification scale height h is 7·5 m.

the centre-line velocity distribution is almost linear. This is consistent with a slug of fluid close to the centre line being retained between the wall and the cylinder and thickening as the cylinder approaches the wall in such a way that lines of constant density remain horizontal and the vertical velocity distribution remains independent of the distance from the cylinder (the régime discussed by Foster & Saffman (1970, § 3)).

Janowitz (1967) obtained solutions for both diffusive and non-diffusive two-dimensional flows about cylinders in an infinite fluid. In the diffusive case, the

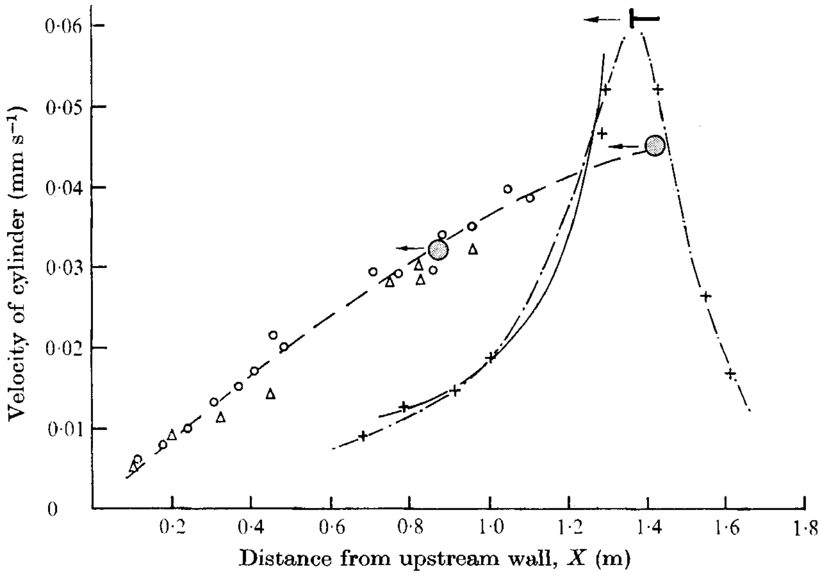

FIGURE 3. The centre-line velocity distribution for (*a*) —○—, circular cylinder of diameter 25·4 mm moving with a velocity of 0·045 mm s^{-1} with $h = 5{\cdot}1$ m; (*b*) ·—+·—, tee-shaped cylinder of height 10 mm moving with a velocity of 0·060 mm s^{-1} with $h = 7{\cdot}5$ m; (*c*) △, circular cylinder of diameter 25·4 mm moving with a velocity of 0·032 mm s^{-1} with $h = 5{\cdot}5$ m; (*d*) ——, Janowitz' equation with a drag per unit length of $3{\cdot}6 \times 10^{-5}$ N m^{-1}.

centre-line velocity distribution u ahead of the cylinder at low speeds and small diffusivities, conditions satisfied by the present experiments, is represented quite closely within a specified range of x by the equation $u = Cx^{-\frac{2}{3}}$. The distance from the body is x and C depends on the stratification scale, the viscosity and diffusivity, the velocity of the cylinder and the drag of the body. If a value of C is chosen to make Janowitz' equation fit experiment (*b*) of figure 3 within the specified range of x then the drag per unit length of the cylinder is found to be $3{\cdot}6 \times 10^{-5}$ N m^{-1}, assuming that the walls are not affecting the results.

Freund & Meyer (1972) present solutions for the diffusive flow, again for the infinite fluid. Using the flow conditions of experiment (*b*) of figure 3, their theory predicts that the drag per unit length is $1{\cdot}84 \times 10^{-4}$ N m^{-1} and that the centre-line velocity 1·34 m ahead of the body is 0·024 mm s^{-1} which means that the theoretical flow velocities are much higher than the experimental ones. It would appear therefore that the wall is attenuating our flow quite considerably.

According to inviscid theory the wavelength of lee waves is 0·33 mm for the conditions of experiment (*b*) of figure 3 and will occur in the region between the body and the position at which the body starts. The wavelength for purely viscous lee 'waves' (Janowitz 1968) is 2·8 mm. The same schlieren system as that used by Stevenson & Thomas (1969) does not show any lee waves in the present experiments. The schlieren photographs of lee waves presented by Stevenson & Thomas were at velocities an order of magnitude higher.

The velocity profiles occurring in these experiments in both the upstream and downstream wakes qualitatively agree with those predicted by the diffusion theories, but the theoretical attenuation of the centre-line velocity given by Freund & Meyer is far less than in the experiments, presumably because of the influence of the upstream and downstream walls. The regime observed in these experiments is to be distinguished from the far higher Reynolds number regime observed by Browand & Winant (1972) to which the non-diffusive theories with their fore and aft asymmetry seem qualitatively relevant (Janowitz 1968, 1971; Graebel 1969).

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